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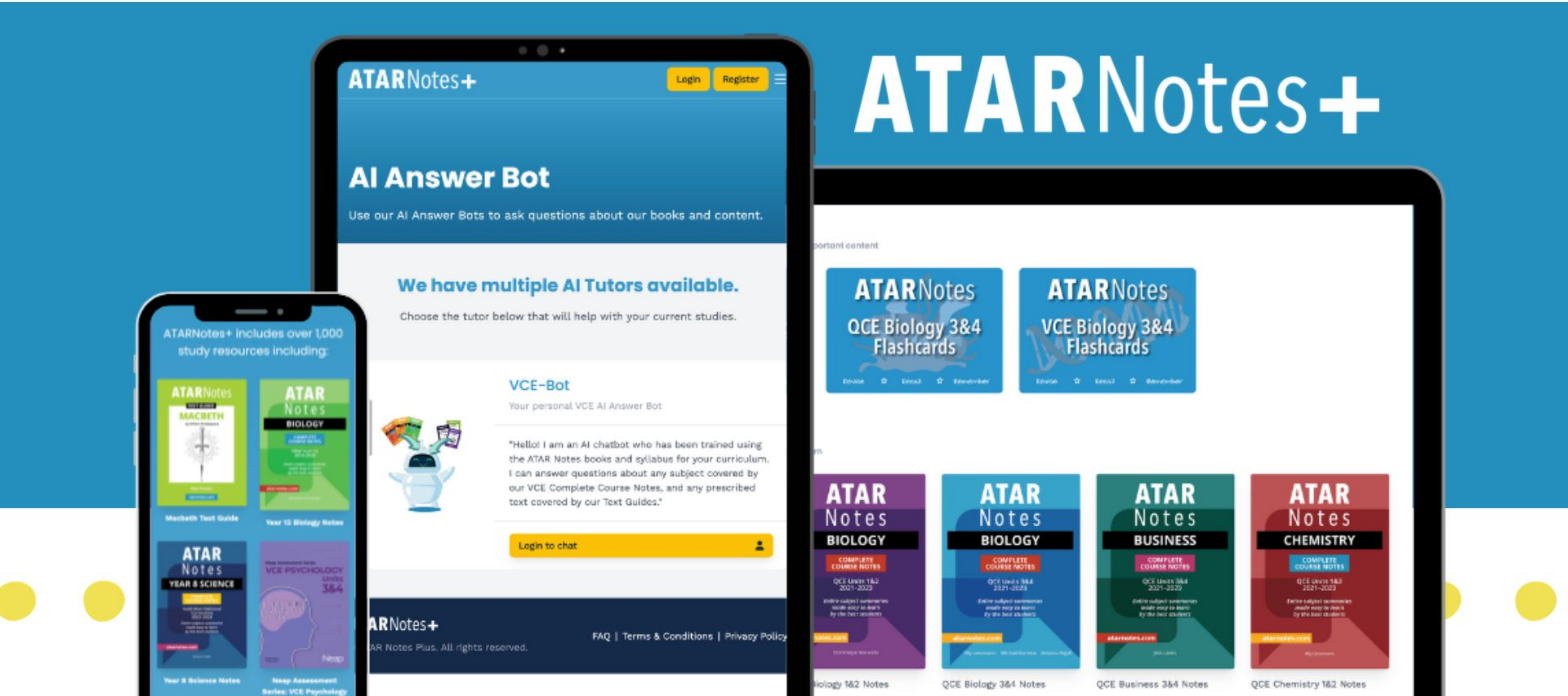

ATAR Notes

Specialist Maths 1&2

ATARNotes January Lecture Series

Presented by:
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About me!

- Hey, everyone my name is Manjot Bhullar
- Bachelor of Biomedical Science
- ATAR of 99.80
- Maths Tutor at Tutesmart
- The subjects I did throughout VCE
    - Chemistry
    - Maths Methods
    - Specialist Maths
    - English
    - Biology
    - Further Maths

Overview

- Specialist Maths is definitely a challenging but rewarding subject
- ½ is quite a bit different to ¾
  - ½ does way more topics and introduces you broadly to all these abstract mathematical concepts
  - ¾ involves focusing in on these key topics and utilising the skills to understand more challenging concepts
- Even if it won’t be assessed in ¾ its really great to have a good understanding of all the topics in ½
- It will provide a neat little insight of further mathematics (pure and applied), but also give you really great skills for ventures in physics, engineering, logic, etc.
   great if you are thinking of doing any engineering or maths
- Gets scaled like crazy.

Overview

Unit 1:

- Proof and number -> broken down into number theory + proof
- Graph theory
- Discrete mathematics

Unit 2:

- Data analysis, probability + stats
- Space+measurement (Trig and vectors)
- Complex numbers
- Functions, relations and graphs

What is Number Theory?

- Number theory is the study of integers! (i.e. the whole numbers {…,-1,0,1,…}.
- Number theory has its biggest application in cryptography and cryptosystems. So, like encrypting your personal data, making safe bank transactions online, etc.
- Some of the stuff in this section will seem like ‘easy’ math, but what is important is to develop the rigor and formality needed for the rest of spesh.

What is Number Theory?

- Hierarchy of numbers
- Include natural numbers (N), Integers (Z), rational numbers(Q), Real numbers (R) and complex numbers (C)

Number Systems

- Natural Numbers (N) : set of positive whole numbers; {1, 2, 3, 4, …}
- Integers (Z) : whole numbers that include positive and negative numbers as well as 0.
- Rational Numbers (Q) : numbers that can be written in the form $\frac{m}{n}$
- Real Numbers (R) : all numbers that are possible on the number line including whole numbers, fractions, surds and continuous decimals
- Complex Number (C) : numbers that do not exist on the real number line ad include both a real and imaginary component

Definition (Factor)

Given two natural numbers, a and b.

a is a **factor** of b if $d \in Z$ such that:

$$ad = b$$

Example (Factors)

Consider two numbers $a = 3$ and $b = 54$. Is a a factor of b?

Definition (Prime)

A natural number, $n > 1$, is prime if its only factors are one and itself.

Example (Prime)

Some examples of prime numbers are 7, 13, 199, etc.

Fundamental theory of Arithmetic

Every natural number, $n > 1$, is either:

1. Prime
2. Can be represented as the product of primes

This representation is unique.

For example, consider the number 156. The smallest prime number that is a factor of 156 is 2:

$$156 \div 2 = 78$$
$$78 \div 2 = 39$$
$$39 \div 3 = 13$$

13 is a prime number. Therefore the unique product of prime numbers to form 156 is: $2 \times 2 \times 3 \times 13$.

Conversion of Rational Numbers

- Finite decimals can be converted to fractions by multiplying the decimal by 10^n where n is the value required to obtain a whole number. Then 10^n is used a divisor and the expression is simplified

$$0.5876 = \frac{0.5876 \times 10^4}{10^4} = \frac{5876}{10000} = \frac{1469}{2500}$$

Conversion of Rational Numbers

Convert $0.\dot{5}3\dot{8}$ *to fraction form.*

1. Multiply the recurring decimal by 10^n where n is the number of digits which recur:

$$0.\dot{5}3\dot{8} \times 10^3 = 538.\dot{5}3\dot{8}$$

2. Subtract the initial recurring decimal to obtain an integer:

$$0.\dot{5}3\dot{8} \times 10^3 - 0.\dot{5}3\dot{8} = 538.\dot{5}3\dot{8} - 0.\dot{5}3\dot{8} = 538$$

3. Divide the integer by $10^n - 1$ to obtain the fraction form:

$$0.\dot{5}3\dot{8}\left(10^3 - 1\right) = 538$$

$$0.\dot{5}3\dot{8} = \frac{538}{10^3 - 1} = \frac{538}{999}$$

What is proof?

- A **mathematical proof** is an argument that demonstrates the absolute truth of a statement.
- Examples of things we might want to prove:
  - There are infinite prime numbers.
  - $1 + 1 = 2$

***54·43.** $\vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54{\cdot}26 . \supset \vdash :. \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

$[*51{\cdot}231] \qquad \equiv . \iota`x \cap \iota`y = \Lambda .$

$[*13{\cdot}12] \qquad \equiv . \alpha \cap \beta = \Lambda \qquad (1)$

$\vdash . (1) . *11{\cdot}11{\cdot}35 . \supset$

$\vdash :. (\exists x, y) . \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda \qquad (2)$

$\vdash . (2) . *11{\cdot}54 . *52{\cdot}1 . \supset \vdash . \text{Prop}$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

  - Don’t worry we’re doing more chill stuff.

Principals for proofs

- Proposition: A statement which is either true or false and is used as a hypothesis in mathematical proofs
- Quantifiers: State the conditions for the proposition.

"for every n such that $n \in \mathbb{N}$, n is an even number"

Conditional Statement

- Before proving anything, lets introduce formally the **conditional.**

Definition (Conditional)

A conditional statement is one of the structure 'if A, then B'. This is written:

$$A \Rightarrow B$$

It also can be verbalised as 'A implies B'

Theorem (Direct Proof)

Say we wish to prove:

$$A \Rightarrow B$$

To do this, we can **assume** A is true, then, we show that B **follows.**

Prove that if w is divisible by 7, w^2 is also divisible by 7.

$$\text{Let } w = 7k_1, \text{ where } k_1 \in \mathbb{Z}:$$

$$\begin{aligned} w^2 &= (7k_1)^2 \\ &= 49{k_1}^2 \\ &= 7\left({k_1}^2\right) \end{aligned}$$

$$\text{as } k_1 \in \mathbb{Z}, \Rightarrow 7k_1^2 \in \mathbb{Z}$$

$$\text{Let } 7{k_1}^2 = k, \text{ where } k \in \mathbb{Z}:$$

$$\therefore w^2 = 7k$$

Definition (Negation)

The **negation** of a statement is the **opposite** of the statement. It is denoted by a ~ or 'not'.

Prove for $a, b \in \mathbb{Z}$ *that* $12a + 18b \neq 2$.

Assume there exists $a, b \in \mathbb{Z}$ such that $12a + 18b = 2$:

$$6(2a + 3b) = 2$$

$$2a + 3b = \frac{1}{3}$$

$$\text{as } a, b \in \mathbb{Z}, \Rightarrow 2a, 3b \in \mathbb{Z}$$

Contradiction: the sum of integers must be an integer

Therefore for $a, b \in \mathbb{Z}$, then $12a + 18b \neq 2$.

Proof by contrapositive

- Sometimes it can be too hard or complicated to use direct proof. So, we prove by using a contrapositive

Theorem (Proof by Contrapositive)

If we wish to prove:

$$A \Rightarrow B$$

We can choose to prove an equivalent statement, known as the **contrapositive:**

$$\sim B \Rightarrow \sim A$$

Proof

Example

Prove that for $a \in \mathbb{Z}$, if $5a + 1$ is an odd number, then a must be an even number.

Contrapositive statement: if a is an odd number, then $5a + 1$ must be an even number.

$$\text{Let } a = 2k_1 + 1 \text{ for } k_1 \in \mathbb{Z}: \qquad \text{(as } a \text{ is an odd number)}$$

$$\begin{aligned} 5a + 1 &= 5(2k_1 + 1) - +1 \\ &= 10k_1 + 5 + 1 \\ &= 10k_1 + 6 \\ &= 2(5k_1 + 3) \end{aligned}$$

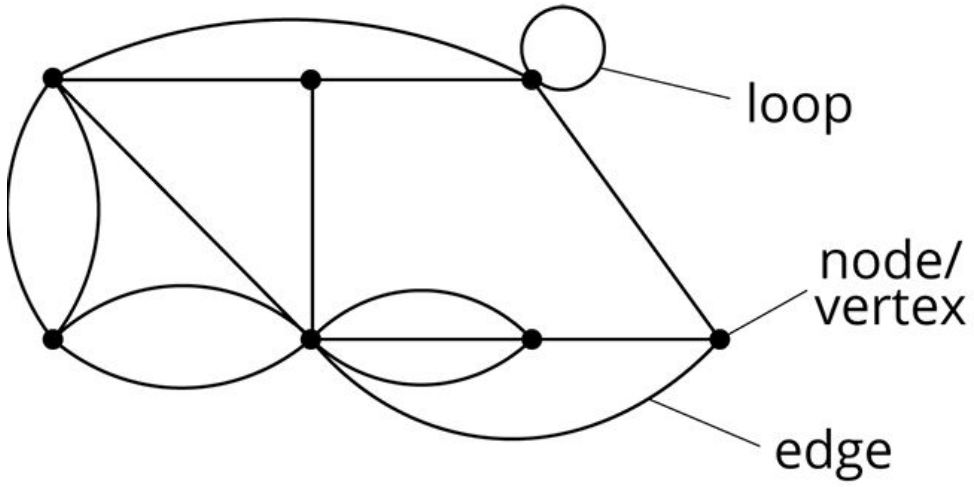
$$\text{Let } k = 5k_1 + 3:$$

$$k = 5k_1 + 3 \in \mathbb{Z} \text{ as } k_1 \in \mathbb{Z}$$

$$\therefore 5a + 1 = 2k, \text{ where } k \in \mathbb{Z}$$

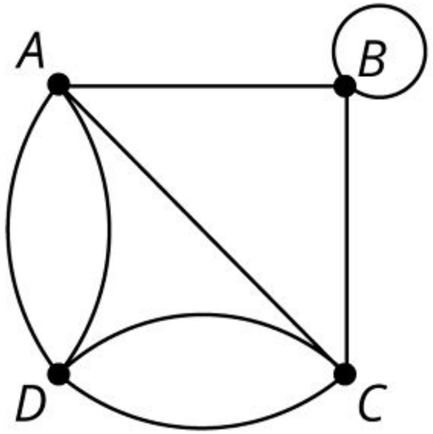
Therefore, if $5a + 1$ is even when a is odd, then $5a + 1$ is odd when a is even.

- A graph or network diagram represents the connections or relationships between objects
  - A node/vertex represents the object
  - Edge represents the connections between the objects



Adjacency Matrix

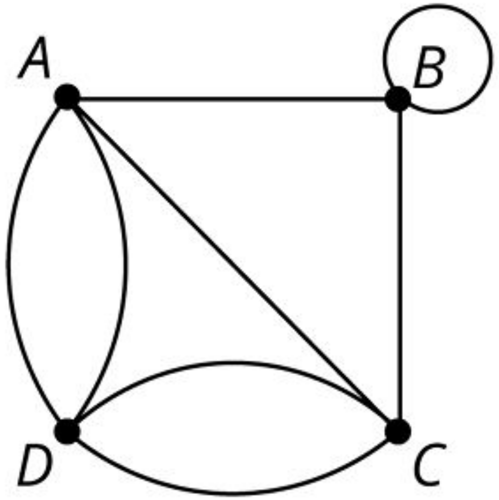
- Displays connections between graphs in matrix form
- Each node is given a letter
- A number is placed in the corresponding row and column of the matrix
- This indicates the number of connections between the assigned nodes



$$
\begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 2 & 0 \end{bmatrix}
\end{array}
$$

Adjacency list

- It has vertices on a column and lists which nodes connect to the specified node



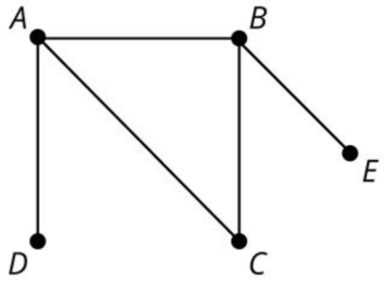
A	→	B	C	D
B	→	A	B	C
C	→	A	B	D
D	→	A	C	

Applications

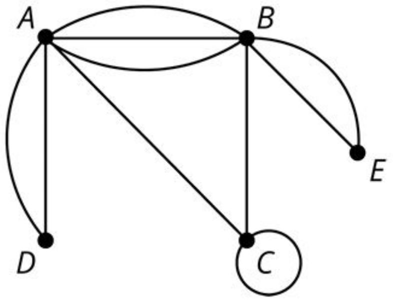
- This is a dot point on the study design so expect it to be assessed!
- Electrical circuits -> each vertex is a node in the circuit, with wires being shown as edges
- Social networks -> individuals/companies/etc can be shown as nodes with edges representing their connections
- Molecules -> vertices represent the atom while edges represent bonds
- Utilitities -> house connection to water companies, etc

A **simple graph** is a graph without any loops or multiple edges between the same vertices.

Simple graph

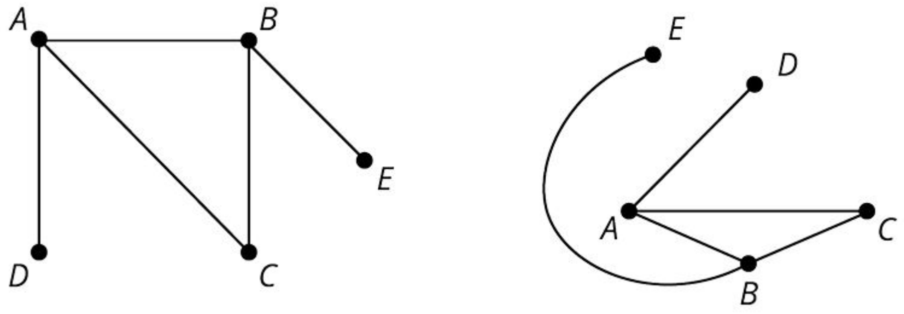


Not a simple graph



An **isomorphic** (or equivalent) graph has the same connectivity between nodes, however the edges or vertices may be arranged differently. The adjacency matrix of each isomorphic graph will be the same.

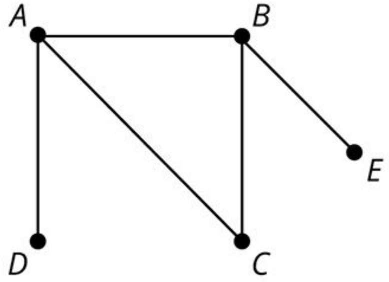
Two isomorphic graphs



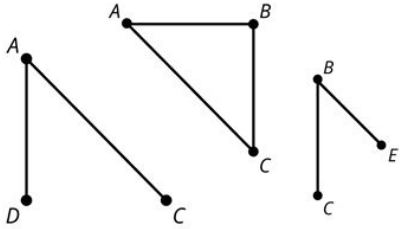
Types of Graphs

A **subgraph** is a part of a larger graph. All the vertices and edges in a subgraph must be a part of a larger graph.

Graph

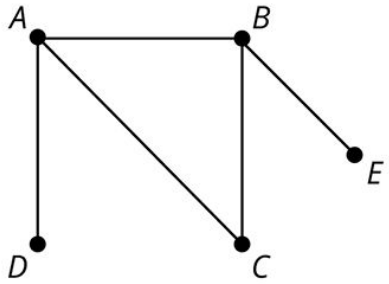


Subgraphs

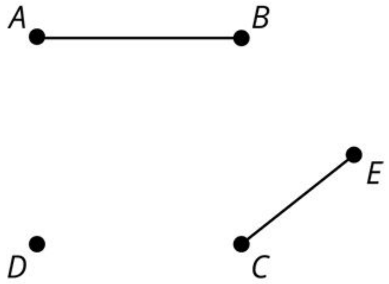


A **connected graph** has every vertex attached to every other vertex, either directly, or indirectly. That is, there are no isolated vertices or isolated subgraphs.

Connected graph



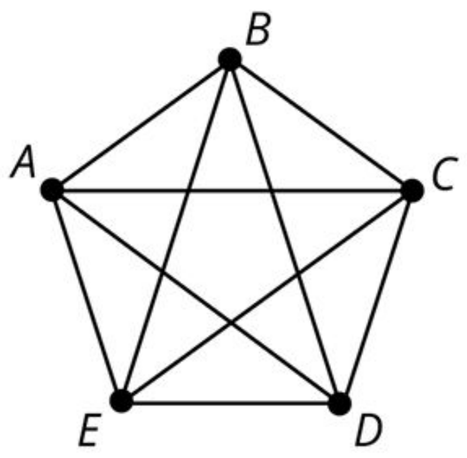
Not a connected graph



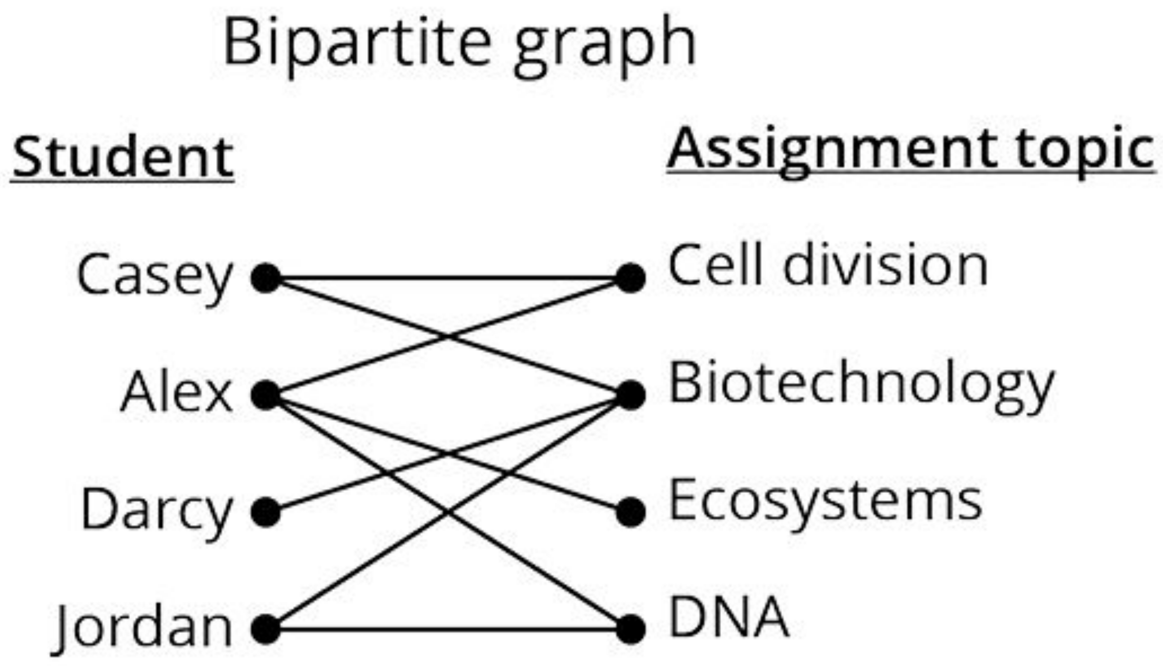
Types of graphs

A **complete graph** is a simple graph with every vertex joined by an edge to every other vertex. There is an edge between every pair of vertices. The complete graph, K_n with n vertices will have $\frac{n(n-1)}{2}$ edges.

Complete graph

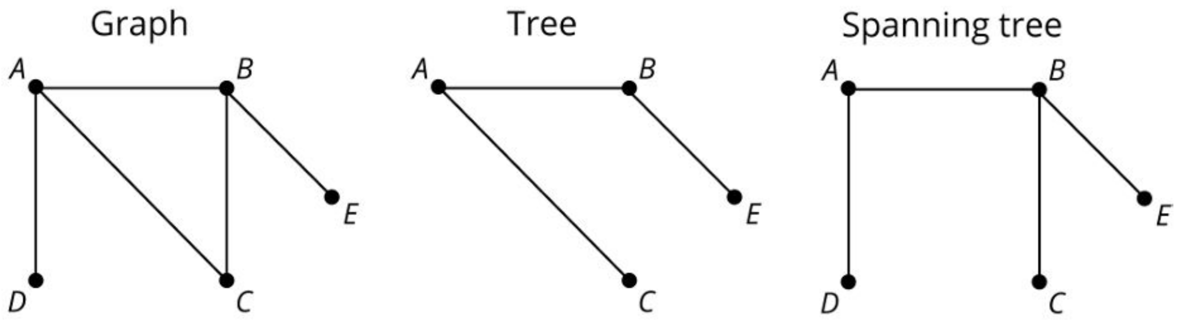


- Graphs which have two separate groups of vertices. Edges in bipartite graphs join a vertex from one group to a vertex from another group



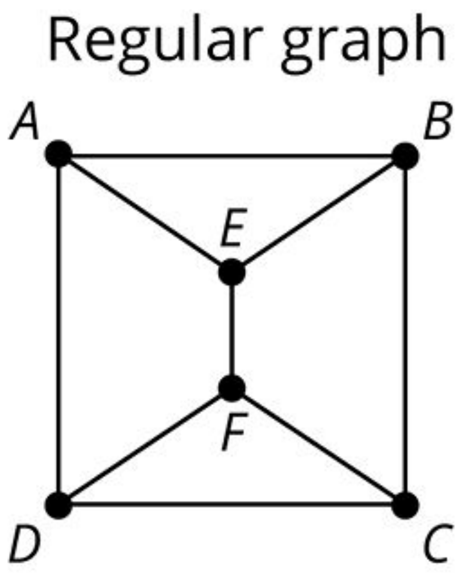
Trees

- A tree is a type of undirected connected graph with no loops, multiple edges or cycles
  - Number of edges in a tree will always be one less that the number of edges (e = v – 1)
- A Spanning tree is a type of tree however it will include all the vertices of the original graph



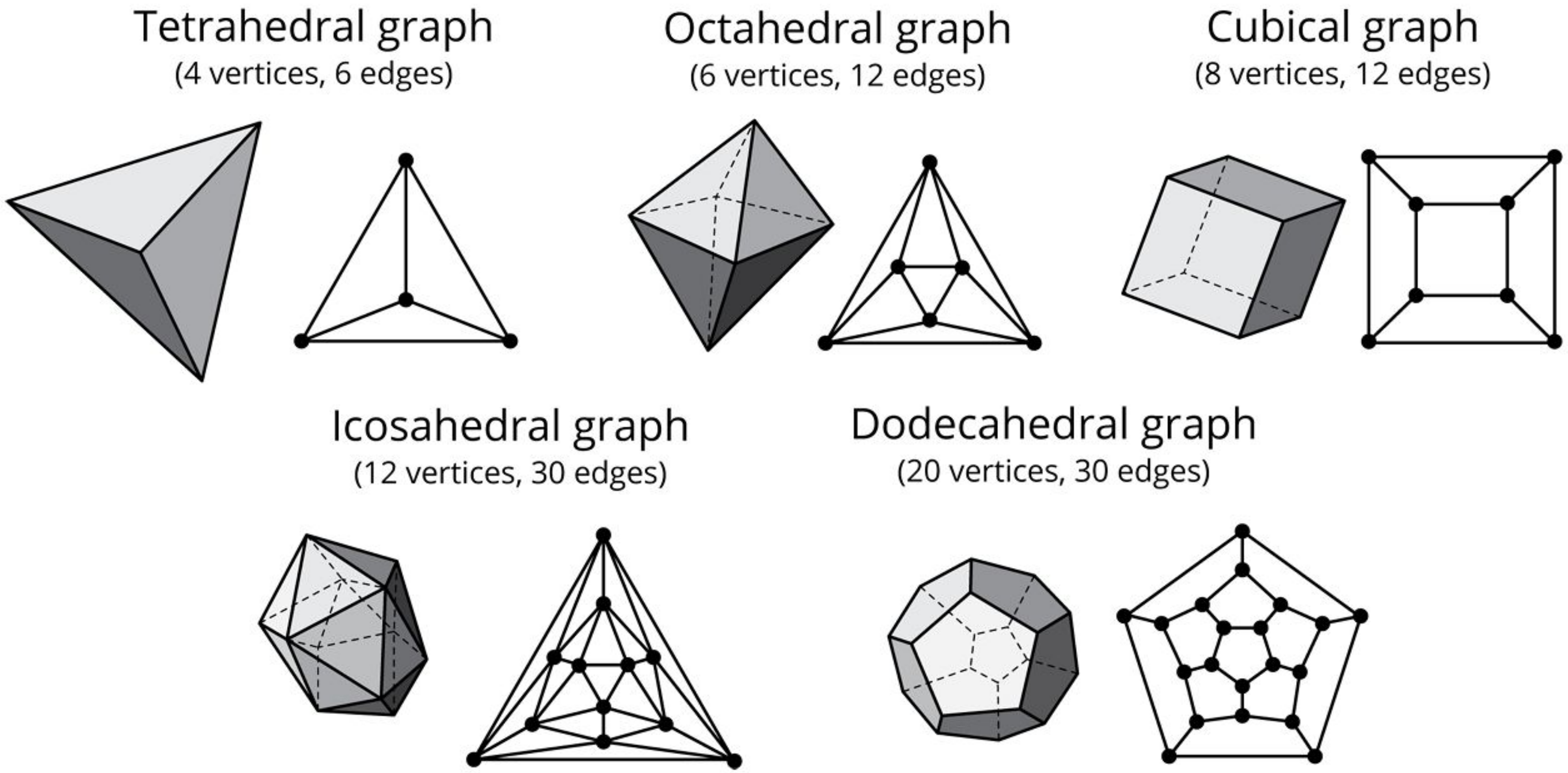
Regular Graph

- A regular graph is a type of graph where each vertex has the same degree.
  - A regular graph where the degree of each vertex is k, is called a k-regular graph



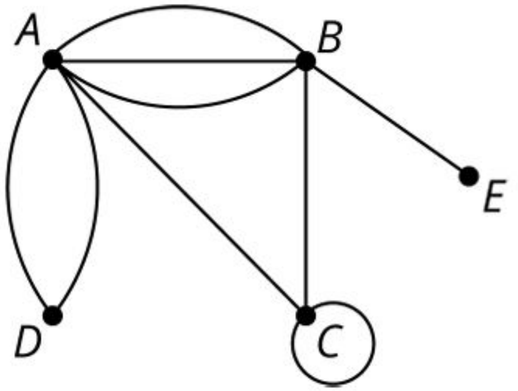
Platonic Graphs

- A type of regular graph which represents the skeleton of a platonic solid and is like the two-dimensional version of the solid



Handshaking Lemma

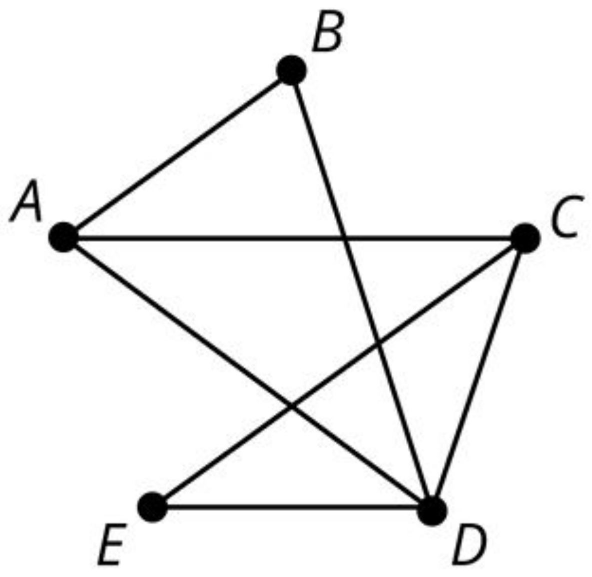
- The degree of a vertex is given by the number of edges that are attached to the vertex. (A loop is considered as two attachments although they are the same edge)

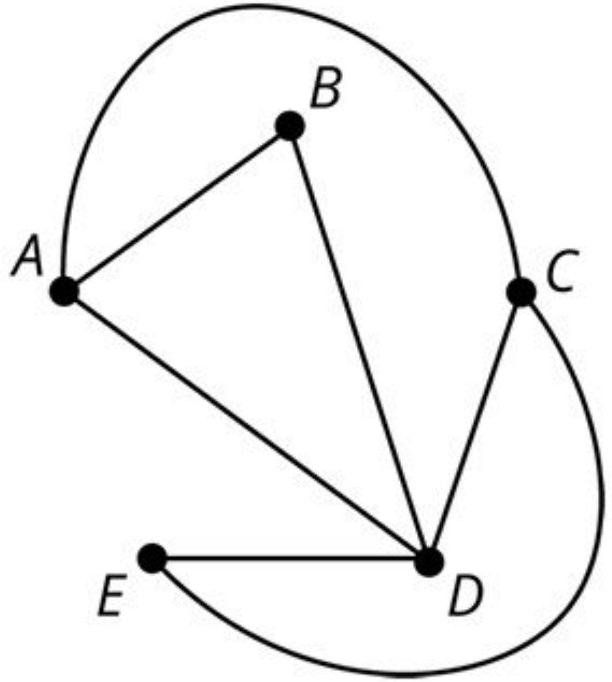


$$\deg(A) = 6$$
$$\deg(B) = 5$$
$$\deg(C) = 4$$
$$\deg(D) = 2$$
$$\deg(E) = 1$$

- The handshaking lemma states that the sum of the vertex degrees is double the number of edges of the graph

- A graph for which an isomorphic graph can be drawn without any edges that overlap

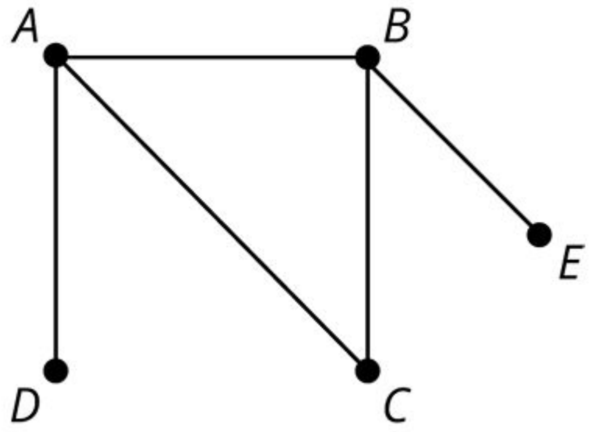




Euler's Formula

- For a planar graph, the number of vertices (v), faces (f), edges (e) follow the relationship v + f - e = 2

Verify Euler's formula from the following graph.



$$v + f - e = 2$$

$$\begin{aligned} \text{LHS} &= v + f - e \\ &= 5 + 2 - 5 = 2 \\ &= \text{RHS} \end{aligned}$$

- Walk -> sequence of edges that links vertices
- Trail -> walk with no repeated edges
- Circuit -> trail which starts and ends at the same vertex
- Path -> walk where no vertices are repeated
- Cycle -> path that starts and ends at the same vertex

Eulerian Trail

- A type of trail in which every edge must be visited exactly once
- Only exists if every vertex has an even degree **except** exactly 2 vertices which have an odd degree

Eulerian Circuit

- Similar to an Eulerian trail but starts and end at the same vertex
- The degree of **all** vertices must be even in order to exist

Hamiltonian Path

- A type of path in which every vertex must be visited exactly once

Hamiltonian Cycle

- Similar to a Hamiltonian path but starts and ends with the same vertex

Summary

Travel term	Use edges more than once	Use vertices more than once	Start and finish at the same vertex	Start and finish at different vertices
Walk	Yes	Yes	No	Yes
Trail	No	Yes	No	Yes
Path	No	No	No	Yes
Circuit	No	Yes	Yes	No
Cycle	No	No	Yes	No

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QUESTIONS?